Coefficient Inverse Problems of Mathematical Physics for Real World Applications

Mikhail V. Klibanov[1], Loc H. Nguyen[1] and Vasily N. Astratov[2]

[1]Department of Mathemarics and Statictics

[2]Department of Physics and Optical Science

Two target categories for the submission: Existing and Emerging Excellence, Unique Distinction.

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Executive Summary. We self-nominate our well federally funded, cutting edge research, world distinguished and truly interdisciplinary group. The group leader is Klibanov. We represent an extremely rare case when mathematicians (Klibanov and Nguyen) have been working for a number of years and on daily basis hands-by-hands with an experimental physicist (Astratov) and engineers on real World applications. Unlike this, mathematicians are usually not involved in experimental studies. Those engineers are Anders Sullivan and Lam Nguyen from the US Army Research Laboratory (ARL). Klibanov, Nguyen, Astratov and ARL collaborate since 2015, and Klibanov collaborates with ARL since 2012. We have total 22 joint publications in 2012-2021 in highly reputable journals, including 10 joint publications with ARL. Klibanov has been consistently well funded by DoD, mostly by US Army Research Office (ARO), in the period of 2005-2021 with the total exceeding $3,300,000. In particular, in the period of 2015-2021 he got total $1,478,440: $1,306,558 from two grants of ARO and $171,882 from Office of Naval Research (ONR). Both Nguyen and Astratov are Co-PIs of the current ARO grant (2018-2021, $690,114), which speaks for itself about the level of our collaborative effort.

We work on Coefficient Inverse Problems (CIPs) of the recovery of a spatially distributed dielectric constant, which is a coefficient of a wave-like Partial Differential Equation (PDE). This PDE governs the propagation of an electromagnetic wave field. To recover that coefficient, measurements of the scattered electromagnetic wave field outside the medium of interest are used. Therefore, if the dielectric constant is computed, the interior structure of the otherwise unknown medium is imaged. We work only with the most economical and most practical data collection scheme of the so-called non redundant data. Straightforward applications, which we pursue on daily basis, are in standoff detection and imaging of land mines and improvised explosive devices (IEDs) as well as in standoff imaging of interiors of buildings. These applications are of a great interest to the Army, see Research Area 3 in the above web page.

CIPs are extremely challenging ones to solve numerically. Nevertheless, we handle them successfully since we are the single team in the world, which uses a globally convergent numerical method for CIPs. This is the <u>revolutionary</u> convexification method of Klibanov. The crucial advantage of the convexification over all other existing numerical methods for CIPs with non-redundant data is that it delivers the correct solution without any advanced knowledge of a good first guess about this solution. All other numerical methods for such CIPs are locally convergent ones since they lead to the correct solution only under an unrealistic assumption of the availability of a good first guess. The reliance on this assumption makes those methods unreliable. Therefore, our group is one of the world's very top ones in the field of Coefficient Inverse Problems.

We believe that the main goal of UNCC investments in our group should be a transformation of our effort from the conventional ARO funding to an internationally recognized Center of Computational Inverse Problems with a corresponding specialization of doctoral students. A possible ARO funding of the above center of excellence would be used as a leverage.

Evidence of Strength and Excellence. Our research team has been focusing on the area of CIPs, which is an interdisciplinary topic among mathematics, physics and engineering. This reseach topic arises from many real world applications; for e.g., detection and identification of land mines and IEDs, nondestructive testing, bio-medical imaging, etc. Indeed, solution of a CIP answers the "diagnostics" question: Given a radiation passing through a medium (e.g. electromagnetic radiation), *how to image the interior structure of that medium, using measurements of that radiation outside the medium*? In particular, we are focused on imaging of buried land mines and targets, which are fully occluded in e.g., a building, see page 8 of the above web page as well as page 5 below about the importance of these topics to the Army. As a result of our achievements in this topic with many publications in highly reputable journals (see the enclosed CVs for some selected ones), our team has attracted the collaboration with ARL. Department of Defense, mostly ARO, has been consistently funding our team from 2005 to present with the total award exceeding \$3,300,000. The total funding in 2015-2021 was \$1,478,440: (1) ARO grant W911NF-15-1-0233, \$616,444 for 2015-2018, (2) ONR grant N00014-15-1-2330, \$171,882 for 2015-2018, (3) ARO grant W911NF1910044, \$690,114 for 2018-2021.

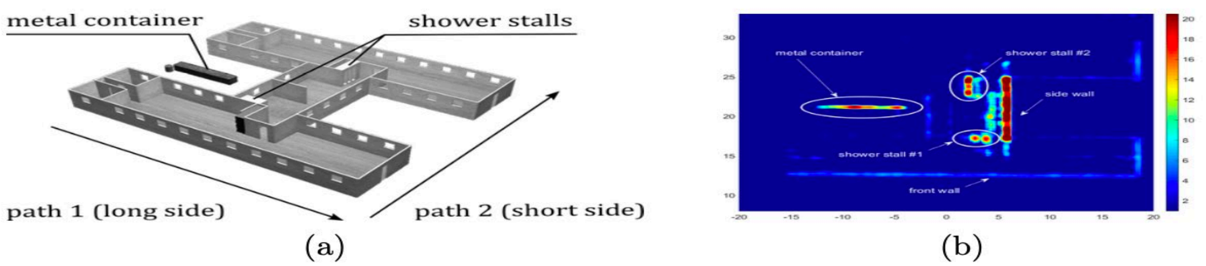

Figure 1. *One of our recent numerical results by the convexification method, see reference number 4 in the CV of Klibanov. a) A schematic image of an inspected building. b) Our image of a). The use of our method can identify some objects inside the building from the external measurements. The experimental data were collected by ARL.*

In mathematics, the above CIP on *how to identify targets fully occluded or buried under the ground* is formulated as the inverse scattering problem. Due to its significance for many applications, the inverse scattering problem has been studied intensively by many researchers worldwide. However, effective algorithms to solve it are very limited. This is especially true for the most economical, the most challenging and the most practically valuable case of non-redundant data collection, outlined above. Being practically oriented, we work only with this case of data collection. The main drawback of all competing algorithms for CIPs with non redundant data is that they are applicable only under some unrealistic mathematical assumptions. The conventional numerical methods for those CIPs are based on the optimization approach. Hence, they suffer from the phenomenon of multiple local minima and ravines, which cannot be handled within the optimization framework. The latter means, in turn that such a method delivers the correct solutions of a CIP only if a good first guess about this solution is available in advance. This, however, is unrealistic and, therefore, unreliable.

Unlike these, the most attractive feature of the convexification method of Klibanov, which is absent in all competing algorithms for CIPs with non redundant data, is that it delivers the correct solution without any advanced knowledge of a first guess about this solution. Currently, the convexification is the only numerical method for CIPs with non redundant data with this property. All other numerical methods for such CIPs are unreliable. The foundational mathematical idea of the convexification is rooted in the seminal publication of A. L. Bukhgeim, M. V. Klibanov, “Global uniqueness of a class of multidimensional inverse problems”, *Soviet Mathematics Doklady, 24, 244-247, 1981*. In this paper, a mathematically elegant and very powerful tool of

Carleman estimates was completely unexpectedly introduced in the field of CIPs. Currently, forty years later, the technique of this paper remains the only one which can work with CIPs with non redundant data. Back in 1981, the publication of this paper has catapulted Klibanov to the world's highest level of experts in Inverse Problems. After that publication, Klibanov has published many papers on the topic of applications of Carleman estimates for CIPs, with many fresh ideas. His latest achievement in this direction is the transformation of the idea of the above cited publication from the pure theory to the convexification principle for numerical studies.

1. Prestigious award: Klibanov was awarded Golden Medal in 2017 for his "Distinguished Impact in Mathematics" by Sobolev Institute of Mathematics of Russian Academy of Science.
2. Roles in the project: We always work hands-by-hands simultaneously in three directions: (1) Modeling and theoretical development (Klibanov), (2) Numerical implementation of the theory of #1 (Nguyen), (3) Confirmations of results of #1&2 on experimental data collected by Astratov and ARL. We believe that this is the most effective way for mathematicians to work on real world applications.
3. Future plans: We plan to significantly improve the current versions of the convexification via moving from the current case of frequency domain data to a more informative case of time domain data. We also plan a further development of nonlinear Synthetic Aperture Radar (SAR) imaging, see Figure 1 and footnote on that page. While the highly popular conventional linear SAR imaging delivers only locations and shapes of targets, we deliver values of dielectric constants as well.
4. Citation impact: Klibanov, Astratov and Nguyen were cited 7159, 6940 and 545 times respectively, according to google scholar. The lesser number of citations of Nguyen is due to the age difference with Klibanov and Astratov.
5. Joint publications: We have total 20 joint publications of three of us, including those published jointly with ARL. 14 of them are joint of three us.
6. Educational Component: We train four doctoral students and one postdoctoral research associate. One student is fully supported and another one is partially supported by the current ARO grant.
7. Diversity: We have a truly diverse team: Loc Nguyen and the postdoc Dr. Vo Khoa are Asian Americans, one of graduate students (Thuy Le) is an Asian female and another student (Ray Abney) is a Native American with a disability.
8. The need of additional resources (see the last paragraph of page 2): Klibanov and Astratov deserve to be awarded distinguished professorships and Nguyen deserves a tenured full professor position. Due to the computationally extensive effort, we need a tenure track mathematician as well as a postdoc with an expertise in computational inverse problems.

Alignment with Regional and National Priorities.

Our research and educational activities are completely aligned with the regional and national priorities. Indeed, here is a statement of page 8 of the web page of ARO https://www.instantmarkets.com/view/ID226132597409103165891151668096413197409

"*In this particular topic call, we restrict our focus to imaging of occluded targets, primarily considering targets that are either buried or fully occluded by walls… Accomplishing these tasks generally involves solving ill-posed computational inverse problems in signal processing from incomplete and noisy data, one of the most challenging problems in computational mathematics*."

We completely cover the whole process of solving Coefficient Inverse Problems. This includes mathematical modeling, developments of cutting-edge mathematical theories of a variety of versions of the convexification methods, numerical implementations of those theories and the successful tests of our algorithms on experimental data of both UNCC and ARL.

We are unaware about another research group of the world that can directly embed their own mathematical theories into the laboratories. The collaboration between Mathematics, Physics and ARL is profound.

As to the education, we truly beleive that given our interesting, important and challenging research topics and consistent federal support, we can recruit good undergraduate, doctoral students and postdoctoral researchers. We plan to arrange a specialization in Computational Inverse Problems.

An important alignent with the regional and national priorities is that we strongly support people from the underrepresented groups. In this regard, we refer to item 7 in section "Evidence of Strength and Excellence".

Supporting documents.

1. The leader of our group is Dr. Mikhail V. Klibanov. His main role is in mathematical modeling and theoretical developments. Klibanov is currently considered as a world leader in the field of Inverse Problems. This area is an important part of the fields of Partial Differential Equations, Numerical Analysis and Scientific Computing. It has many realistic applications and we focus on those of the interest to the Army. The above cited publication of 1981 of A.L. Bukhgeim and M.V. Klibanov was the first one which has discovered a unified method for rigorous proofs of the global uniqueness theorems for a wide class of Coefficient Inverse Problems with non-redundant data. Until now this method remains both the most general and the most powerful one for CIPs with non redundant data. More recently Klibanov has transformed the purely theoretical idea of that paper in the revolutionary convexification numerical method, which is the core of the research activity of our group. During his career, Klibanov has made many pioneering contributions, both theoretical and numerical ones, to the field of Inverse Problems. Most of them are based on Carleman estimates. A clear proof of this is that Dr. Klibanov was awarded in 2017 a Golden Medal for his "Distinguished impact in mathematics" by the Sobolev Institute for Mathematics of the Russian Academy of Science, which is one of the World strongest mathematical entities. At UNCC, he has developed our interdisciplinary research team and has been supported for the last sixteen years (since 2005) by a number of federal grants, mostly from ARO, with the total amount exceeding $3,300, 000.

2. The role of Dr. Loc H. Nguyen is in numerical analysis and computational implementations of algorithms, which are theoretically developed by Klibanov. In addition, Nguyen often modifies Klibanov's theories. Nguyen is instrumental in our team due to his expertise in numerical methods of applied mathematics. Dr. Nguyen plays a key role due to his innovative ideas and programming skills. In the recent years, Dr. Nguyen analytically and numerically solved a series of inverse source problems. In addition, he has also developed, independently of Klibanov, some new modifications of the convexification method to solve several nonlinear inverse problems. Some combinations of numerical methods of Klibanov and Nguyen will be used to solve many more important Coefficient Inverse Problems of our project. Klibanov and Nguyen are responsible supervise doctoral and undergraduate students and postdoctoral researchers in the mathematical sub-team.
3. Dr. Astratov is responsible for collection of microwave experimental data, which we use to verify the performance of our algorithms. Dr. Astratov is a pioneer of the opal photonic crystals, and one of the Co-PIs at the Center on Metamaterials of UNCC. Dr. Astratov is the head of the microwave laboratory located in the Grigg Hall of UNCC. This laboratory was created several years ago by the support of an ARO grant obtained by Dr. Klibanov. The goal of the laboratory is to collect experimental data for our research group. Dr. Astratov is supervising two doctoral students and two undergraduate students. One of those doctoral students, Mr. Grant Bidney, is working with Dr. Astratov to collect those experimental data. Bidney is fully supported by the current ARO grant where PI is Klibanov and Co-PIs are Astratov and Nguyen.

Klibanov is the PI and Nguyen and Astratov are two Co-PIs of the above mentioned proposal for the Center of Excellence of ARO. Below is the brief description of the contributions of the above three faculty of this self-nomination.

Michael Klibanov	Full Professor	Will be responsible for the mathematical modeling and theoretical guidance to the team. Will oversee the overall performance of the group
Loc Nguyen	Assistant Professor	Work with Dr. Klibanov on theoretical studies. He will also guide the students from the computational standpoint and will work on the computations.
Vasily Astratov	Full Professor	Operate and supervise the students to operate our the microwave device to collect the experimental data.